DUALITIES OF MODULAR LATTICES

BY

MARION PRESTON EMERSON

B.S., Southwest Missouri State Teachers College, 1938
M.S., University of Wisconsin, 1948

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICS
IN THE GRADUATE COLLEGE OF THE
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DUALITIES OF MODULAR LATTICES

A permutation of the elements of a partially ordered set that reverses the order relations is a duality. The pole-polar relation of elementary geometry and the well known dualities of the lattice of subgroups of a finite abelian group are classical examples. Analogous to the study of the fixed points of projectivities (and other transformations) we investigate the absolute (self conjugate) elements of dualities. The absolute elements of a duality are those elements that are contained in or contain their image. An element is lower absolute when it is contained in its image. The significance of absolute elements is exemplified in von Staudt's definition of the conics as the totality of the absolute points of a polarity.

The lower bound O and the upper bound I are absolute under all dualities of a lattice; however, these elements may be the only absolute elements of a duality. For example, the lattice of sub-sets of a finite set S has a duality defined by mapping each sub-set on its complement. The only absolute elements under this duality are the null set and S.

It has been shown that every duality of a finite projective plane has at least one absolute point. The generalization of this theorem is the basic problem of this paper. By introducing the concepts of a *peak,* the sum of the non-complemented points, a *crater,* the product of the non-complemented hyperplanes, and their generalizations, we reduce our problem to that of investigating complemented lattices.

An indecomposable complemented lattice is a projective geometry. Hence, in order to reduce our problem to that of investigating projective geometries, we consider the effect of dualities on direct decompositions of lattices. Sufficient conditions are given to determine the absolute elements of a lattice from the absolute elements of the direct components. We generalize the theorem that every duality of a finite projective plane has at least one absolute point by showing that a finite projective geometry of dimension d has at least as many absolute points as there are points carried by any of its (d-2)-dimensional subspaces.

VITA

Marion Preston Emerson was born February 24, 1918, in Washburn, Missouri. He was graduated from Washburn High School in 1934. He attended Southwest Missouri State Teachers College where he received the degree of Bachelor of Science in 1938. He taught for three years in Eureka High School, Eureka, Missouri. He served for four years in the armed forces of the United States and in 1946 he was discharged as a captain from the Army Air Force. He attended the University of Wisconsin where he received the degree of Master of Science in 1948. He held a graduate assistantship at that institution and also at the University of Illinois where he continued his graduate study. He is a member of Kappa Mu Epsilon, Pi Mu Epsilon, and Sigma Xi.